Homework #7 (100 points) - Show all work on the following problems:

(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (30 points): Consider a circular ring in the x-y plane with radius R, centered at the origin, carrying a uniform linear charge density λ. Find the first three terms (n =0, 1,2 = monopole, dipole, quadrupole) in the multipole expansion for $V(r,\theta)$. *Hints: Note that in this case the 3-d integral of the volume charge density in Eq. 3.95 is replaced by a 1-d integral over the linear charge density λ. Start by expressing the angle α in terms of the spherical coordinate angles θ, ϕ for the position vector and θ', ϕ' for the source vector.*

Problem 2 (10 points): Find the dipole moment of a spherical shell of radius R with a surface charge $\sigma = k\cos(\theta)$.

Problem 3 (20 points): Show that the electric field of a perfect dipole (Eq. 3.103) can be expressed in the coordinate-free form:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}]$$

Problem 4 (20 points): Show that the energy of an ideal dipole $\vec{p}$ in an electric field $\vec{E}$ is given by $U = -\vec{p}\cdot\vec{E}$. *Hint: Consider the energy required to assemble the dipole, when bringing the charges from infinity.*

Problem 5 (20 points): Consider a sphere of radius R centered at the origin, with a polarization $\vec{P}(\vec{r}) = k\vec{r}$.

5a (10 points): Find the bound volume and surface charge densities ρ_b and σ_b.

5b (10 points): Find the electric field inside and outside the sphere.